Math 113 homework due 1/23

Go to the roots, of these calculations! Group the operations. Classify them according to their complexities rather than their appearances! This, I believe, is the mission of future mathematicians

– Évariste Galois[1]

(1) Read and review sections 12.1 and 12.2 in the book and the supplemental reading.

(2) For this question we will work in D_{10}, the group of symmetries of the pentagon. The standard form for these symmetries that we established in class is given in the following list:

$$I, R, R^2, R^3, R^4, F, FR, FR^2, FR^3, FR^4$$

Use the relations $R^5 = I$, $F^2 = I$, and $RF = FR^4$ to write each of the following in one of the standard forms.
(a) FRF, (b) $(R^2)(R^4)(R^3)$, (c) $(FR)(R^2)(FR)(FR)$, (d) $RFRF$

(3) Show that D_8, the group of symmetries of the square, is generated by R_{90} (rotation by 90 degrees)and F (vertical flip).

(4) Recall that Z_6 is the group of rotations of the hexagon. It has elements I, R, R^2, R^3, R^4, R^5 and the single relation $R^6 = I$. The element R clearly generates this group (meaning every element is obtained by combining R with itself).
(a) Does R^2 generate the group?
(b) Does R^3 generate the group?
(c) Does R^5 generate the group?
In each case, explain why or why not.

(5) Question 11.9 Hint: there is an easy way to do this!

From the reading on the classification of finite simple groups:

(6) Show that there is no subgroup of D_8 that is structurally the same as C_3.

(7) According to Ewes, what are the roles of abstraction and classification in mathematics? Why are they important? Do you agree?

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[1] I was tempted to include this quote from Galois instead, but thought it insufficiently inspiring: *"Genius is condemned by a malicious social organization to an eternal denial of justice in favor of fawning mediocrity."* Though he was a brilliant mathematician and founded much of modern group theory, Galois had a tough life, a fierce temper (often aimed at his professors!) and an unfortunate death.